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# Probabilistic Behaviour of A Redundant Complex System with Imperfect Switching, Environmental Common Cause and Human Error Effects under Head-Of-Line Repair Discipline

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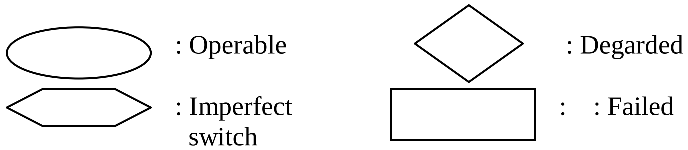
**ABSTRACT**
Series-parallel systems are made up of combination several series & parallel configuration to obtain system reliability down into homogeneous subsystems. This is very simple to analyse a simple series-parallel system without any error effects, But very difficult to analyse a complex system with various error effects. This paper we presented mathematical analysis of a Redundant Complex System with Imperfect switching, Environmental and Common Cause and Human error effects under Head of Line Repair Discipline.
**Keywords:** Imperfect switching, Reliability, Availability, MTTF, State Transition diagram.

## I. INTRODUCTION

Kontoleon & Konoleon [2] had considered a system subject to partial and catastrophic failures to be repaired at a single service station and also assumed exponential distribution for repair. Gupta and Mittal [3] considered a standby redundant system and two types of failures to be repaired at a single service station with general repair time distribution under pre-emptive resume repair Discipline incorporating environmental effects. Further, Gupta and Agarwal [1] developed a model with two types of failure under general repair time distribution and under different repair discipline. Consequently Agarwal, Mittal *et. al.,* [4] have solved the model of a Parallel redundant complex system with two types of failures with Environmental effect under preemptive Resume repair Discipline. Mittal, Gupta *et. al.,* [1,3] have assumed constant repair rate due to environmental failure and perfect switching over device of a standby redundant system under pre-emptive resume and repeat discipline respectively. But it is not always possible that the switch is perfect. It may have some probability of failure. So the authors have initiated the study keeping in mind the practical aspect of imperfect switch. Also repair due to environmental effects is considered to follow general time distribution.

In this paper, we consider a complex system consisting of three independent, repairable subsystems A, B & C. Subsystem B is comprised of two identical units $B_1$ & $B_2$. The subsystem C is in standby redundancy to be switched into operation when both the identical units $B_1$ & $B_2$ of subsystem B fails, through an imperfect switching over device. For the smooth operation of the system, the subsystems B or C are of vital importance. The failure of the subsystem C results into non-operative state of the system. At the time of installation, the units of subsystem B have the same failure rate but due to adverse environmental effects, the failure rate of the subsystem C increases by the time, it is switched into operation. Subsystem A has minor failure and subsystem C has major and minor failures. Minor failure reduces the efficiency of the system causing a degraded state while major failure results into a non-operable state of the system. The whole complex system can also be in a total failure state due to Human error or Environmental effects like temperature, humidity etc. The repair rate due to Human error or Environmental failure is considered general. Laplace transforms of the time dependent probabilities of the system being in various states have been obtained by employing the techniques using supplementary variable under Head-of line Repair Discipline. The general solution of the problem is used to evaluate the ergodic behaviour and some particular case. At the end of the chapter some numerical illustrations are given and various reliability parameters have been calculated. The flow of states is depicted in the state transition diagram.

The following symbols are used in state transition diagram:



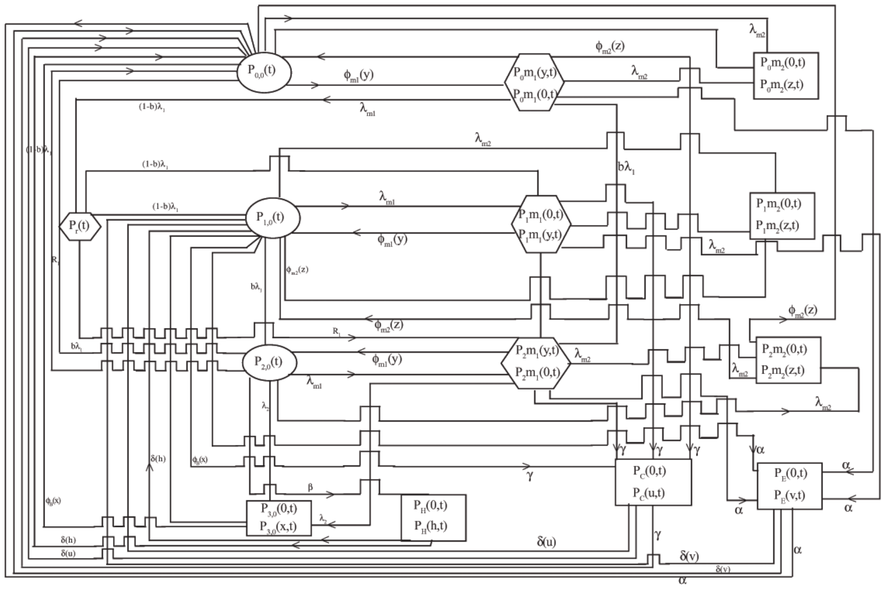




**Fig. 1. STATE TRANSITION DIAGRAM**

## II. NOTATIONS:

$\lambda_1$ : Minor failure rate of the operating units $B_1$ & $B_2$ of subsystem B.

$\lambda_2(\lambda_1<\lambda_2)$: Minor failure rate of the operating unit of subsystem C

$\lambda_{m_1}$ : Minor failure of subsystem A.

$\lambda_{m_2}$ : Major failure of subsystem B & C.

x: Elapsed repair time for both units of subsystem B.

y: Elapsed minor repair time for the subsystem A, B & C.

z: Elapsed major repair time for the subsystem B & C.

v: Elapsed repair time for environmental failure.

h: Elapsed repair time for human error failure.

$\phi_B(x)$ : Repair rate of subsystem B

$\phi_{m_1}(y)$ : Repair rate of minor failure of subsystems A, B & C.

$\phi_{m_2}(z)$ : Repair rate of major failure of subsystems B & C.

$\alpha$ : Failure due to environmental effects.

$\beta$ : Failure due to human error.

$\gamma$ : Failure due to common cause.

$\delta(v)$ : Repair rate due to environmental effects.

$\delta(h)$ : Repair rate due to Human error.

$\delta(c)$ : Repair rate due to common cause

$b$, $R_1$ Probability of the successful operation of the switching over device

and constant repair rate of switching over device.

## III. ASSUMPTIONS

(1) Initially, at time t = 0 the system is in operable state, i.e., it operates in its normal efficiency state
(2) The subsystem B consists of two identical units $B_1$ & $B_2$.
(3) Switching device is imperfect.
(4) When the system starts functioning the subsystems A & B will operate and C is in stand by.
(5) When the system starts functioning both the subsystems B & C have the same failure rate $\lambda_1$ as minor failure, but as the time passes due to the adverse environmental effects, the failure rate of the stand by unit i.e., subsystems C increase to $\lambda_2$ which is the minor failure of subsystem C by the time it is needed to operate.
(6) Subsystem A has no major failures.
(7) During the degraded state of the system due to minor failure in subsystems A, B & C, major failures may also occur in subsystems B & C.
(8) All the failures are distributed exponentially.
(9) All the repair follows general time distribution.

(10) The system fails completely due to environmental effects or due to major failures of the subsystems B & C or due to the failure caused by Human error.
(11) Repair of the subsystem C is taken only in failed state.
(12) During the repair of the subsystem C in failed state, the failed units of subsystem B are also repaired.

**STATE PROBABILITIES DESCRIPTION:**

(i) $P_{a,0}(t)$ : Probability that the system is in operable state at the time t, where a=0, 0, 1, 2.

(ii) $P_{3,0}(x,t)$: Probability that the system is the failed state and is under repair with elapsed repair time in the interval (x, x+Δ).

(iii) $P_{a,m_1}(y,t)$: Probability that the system is in degraded state and is under repair with elapsed repair time lying in the interval (y, y+Δ) where a = 0,1,2.

(iv) $P_{a,m_2}(z,t)$ : Probability that the system is the failed state and is under repair with elapsed repair time in the interval (z, z+Δ) where a = 0,1,2.

(v) $P_E(v,t)$: Probability that the system is in failed state due to Environmental failure and is under repair with elapsed Repair time in the interval (v, v+Δ).

(vi) $P_r(t)$: Probability that the system is in failed state due to failure of the switch in switching the subsystem C when subsystem B completely fails at any time t.

(vii) $P_h(h,t)$: Probability of the system is in failed state due to Human error and is under repair with elapsed repair time in the interval (h, h+Δ).

(viii) $P_v(u,t)$: Probability of by the system is in failed state due to common cause error and is under repair with elapsed repair time in the interval (u, u+Δ).

## IV. FORMULATION OF THE MATHEMATICAL MODEL:

Using continuity arguments and probability considerations, we obtain the following difference differential equations governing the stochastic behaviour of the complex system, which is discrete is space and continuous in time:

$$\left[\frac{d}{dt}+\lambda_{m_2}+\lambda_{m_1}+\lambda_1+\alpha+\gamma\right]P_{0,0}(t)=\int_0^\infty P_{3,0}(x,t)\phi_B(x)dx$$
$$+\int_0^\infty P_{0,m_1}(y,t)\phi_{m_1}(y)dy+\int_0^\infty P_{0,m_2}(z,t)\phi_{m_2}(z)dz$$
$$+\int_0^\infty P_E(v,t)\delta(v)dx+\int_0^\infty P_{2,m_2}(z,t)\phi_{m_2}(z)dz+\int_0^\infty P_H(h,t)\delta(h)dx \tag{1}$$
$$+\int_0^\infty P_C(u,t)\delta(u)dx$$

$$\left[\frac{d}{dt}+\lambda_{m_2}+\lambda_{m_1}+\lambda_1+\alpha+\gamma\right]P_{1,0}(t)=\int_0^\infty P_{3,0}(x,t)\phi_B(x)dx$$
$$+\int_0^\infty P_{1,m_1}(y,t)\phi_{m_1}(y)dy+\int_0^\infty P_{1,m_2}(z,t)\phi_{m_2}(z)dx$$
$$+\int_0^\infty P_E(v,t)\delta(v)dx+\int_0^\infty P_{2,m_2}(z,t)\phi_{m_2}(z)dz+\int_0^\infty P_H(h,t)\delta(h)dx \tag{2}$$
$$+\int_0^\infty P_C(u,t)\delta(u)dx$$

$$\left[\frac{d}{dt}+\lambda_{m_1}+\lambda_{m_2}+\lambda_2+\alpha+\beta+\gamma\right]P_{2,0}(t)=b\lambda_1P_{0,0}(t)+b\lambda_1P_{1,0}(t)$$
$$+R_1P_r(t)+\int_0^\infty P_{2,m_1}(y,t)\phi_{m_1}(y)dy \tag{3}$$

$$\left[\frac{d}{dt}+2R_1\right]P_r(t)=(1-b)^{\lambda_1}\left\{\begin{array}{l}P_{0,0}(t)+b\lambda_1P_{0,0}(t)+\int_0^\infty P_{0,m_1}(y,t)dy+P_{1,0}(t)\\+\int_0^\infty P_{1,m_1}(y,t)dy\end{array}\right\} \tag{4}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\phi_B(x)\right]P_{3,0}(x,t)=0 \tag{5}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_{m_2}+\lambda_1+\alpha+\gamma+\phi_{m_1}(y)\right]P_{0,m_1}(y,t)=0 \tag{6}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_{m_2}+\lambda_1+\alpha+\gamma+\phi_{m_1}(y)\right]P_{1,m_1}(y,t)=0 \tag{7}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_{m_2}+\lambda_2+\alpha+\gamma+\phi_{m_1}(y)\right]P_{2,m_1}(y,t)=0 \tag{8}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\phi_{m_2}(z)\right]P_{0,m_2}(z,t)=0 \tag{9}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\phi_{m_2}(z)\right]P_{1,m_2}(z,t)=0 \tag{10}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+2\phi_{m_2}(z)\right]P_{2,m_2}(z,t)=0 \tag{11}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial h}+2\delta(h)\right]P_H(h,t)=0 \tag{12}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial v}+2\delta(v)\right]P_E(v,t)=0 \tag{13}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial u}+2\delta(u)\right]P_C(u,t)=0 \tag{14}$$

**BOUNDARY CONDITIONS**

$$P_{3,0}(0,t)=\lambda_2P_{2,0}(t)+\int_0^\infty \lambda_2P_{2,m_1}(y,t)dy \tag{15}$$

$$P_{0,m_1}(0,t)=\lambda_{m_1}P_{0,0}(t) \tag{16}$$

$$P_{1,m_1}(0,t)=\lambda_{m_1}P_{1,0}(t) \tag{17}$$

$$P_{2,m_1}(0,t)=\lambda_{m_2}P_{2,0}(t)+b\lambda_1\int_0^\infty P_{0,m_1}(y,t)dy+b\lambda_1\int_0^\infty P_{1,m_1}(y,t)dy+R_1P_r(t) \tag{18}$$

$$P_{0,m_2}(0,t)=\lambda_{m_2}P_{0,0}(t)+\lambda_{m_2}\int_0^\infty P_{0,m_1}(y,t)dy \tag{19}$$

$$P_{1,m_2}(0,t) = \lambda_{m_2} P_{1,0}(t) + \lambda_{m_2} \int_0^\infty P_{1,m_1}(y,t)dy \tag{20}$$

$$P_{2,m_2}(0,t) = \lambda_{m_2} P_{2,0}(t) + \lambda_{m_2} \int_0^\infty P_{2,m_1}(y,t)dy \tag{21}$$

$$\begin{aligned} P_E(0,t) &= \alpha P_{0,0}(t) + \alpha P_{1,0}(t) + \alpha P_{2,0}(t) + \alpha \int_0^\infty P_{0,m_1}(y,t)dy \\ &\quad + \alpha \int_0^\infty P_{1,m_1}(y,t)dy + \alpha \int_0^\infty P_{2,m_1}(y,t)dy \end{aligned} \tag{22}$$

$$\begin{aligned} P_C(0,t) &= \gamma P_{0,0}(t) = \gamma P_{1,0}(t) + \gamma P_{2,0}(t) + \gamma \int_0^\infty P_{0,m_1}(y,t)dy \\ &\quad + \gamma \int_0^\infty P_{1,m_1}(y,t)dy + \gamma \int_0^\infty P_{2,m_1}(y,t)dy \end{aligned} \tag{23}$$

$$P_H(0,t) = \beta P_{2,0}(t) \tag{24}$$

**INITIAL CONDITIONS**

$P_{0,0}(0)$ = 1 and all other states probabilities are zero at time t = 0.

from 6

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \lambda_{m_2} + \lambda_1 + \alpha + \gamma + \phi_{m_1}(y)\right] P_{0,m_1}(y,t) = 0$$

Taking Laplace transforms on both sides

$$\frac{\frac{\partial}{\partial y} P_{0,m_1}(y,s)}{P_{0,m_1}(y,s)} = -\left[S + \frac{\partial}{\partial y} + \lambda_{m_2} + \lambda_1 + \alpha + \gamma + \phi_{m_1}(y)\right]$$

Integrating ‘y’ b/w the limits ‘0’ to ‘y’

$$\Rightarrow P_{0,m_1}(y,s) = P_{0,m_1}(0,s) e^{-[s+\lambda_{m_2}+\lambda_1+\alpha+\gamma]y - \int_0^y \phi_{m_1}(y)dy}$$

from equation (16)

$$P_{0,m_1}(0,t) = \lambda_{m_1} P_{0,0}(t)$$

Taking Laplace transforms on both sides

$$\Rightarrow P_{0,m_1}(y,s) = \lambda_{m_1} P_{0,0}(s) e^{-[s+\lambda_{m_2}+\lambda_1+\alpha+\gamma]y - \int_0^y \phi_{m_1}(y)y}$$

On simplification

$$P_{0,m_1}(y,s) = \lambda_{m_1} P_{0,0}(s) V_1(s) \tag{25}$$

where $V_1(s) = \dfrac{1 - s_{m_1}(s + \lambda_{m_2} + \lambda_1 + \alpha + \gamma)}{s + \lambda_{m_2} + \lambda_1 + \alpha + \gamma}$

from equation (7)

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \lambda_{m_2} + \lambda_1 + \alpha + \gamma + \phi_{m_1}(y)\right] P_{1,m_1}(y,t) = 0$$

Taking Laplace transforms on both sides

$$\Rightarrow \frac{\frac{\partial}{\partial y} P_{1,m_1}(y,s)}{P_{1,m_1}(y,s)} = -\left[s + \lambda_{m_2} + \lambda_1 + \alpha + \gamma + \phi_{m_1}(y)\right]$$

Integrating 'y' between the limits '0' to 'y'

$$\Rightarrow P_{1,m_1}(y,s) = P_{1,m_1}(0,s) e^{-[s+\lambda_{m_2}+\lambda_1+\alpha+\gamma]y - \int_0^y \phi_{m_1}(y)dy}$$

from equation 16

$$P_{0,m_1}(0,t) = \lambda_{m_1} P_{1,0}(t)$$

Taking Laplace transforms on both sides

$$P_{1,m_1}(0,s) = \lambda_{m_1} P_{1,0}(s)$$

$$\Rightarrow P_{1,m_1}(y,s) = \lambda_{m_1} P_{1,0}(s) e^{-[s+\lambda_{m_2}+\lambda_1+\alpha+\gamma]y - \int_0^y \phi_{m_1}(y)dy}$$

$$P_{1,m_1}(y,s) = \lambda_{m_1} P_{1,0}(s) V_1(s) \tag{26}$$

From equation 4

$$\left[\frac{d}{dt} + 2R_1\right] P_r(t) = (1-b)\lambda_1 \left\{ \begin{array}{l} P_{0,0}(t) + \int_0^\infty P_{0,m_1}(y,t)dy + P_{1,0}(t) \\ + \int_0^\infty P_{1,m_1}(y,t)dy \end{array} \right\}$$

Taking Laplace transforms on both sides

$$\Rightarrow P_r(s) = \frac{(1-b)\lambda_1}{s+2R_1} \left\{ \begin{array}{l} P_{0,0}(s) + \int_0^\infty P_{0,m_1}(y,s)dy + P_{1,0}(s) \\ + \int_0^\infty P_{1,m_1}(y,s)dy \end{array} \right\}$$

$$= \frac{(1-b)\lambda_1}{s+2R_1} [(1 + \lambda_{m_1} V_1(s) P_{0,0}(s) + (1 + \lambda_{m_1} V_1(s) P_{0,0}(s)]$$

$$= \frac{(1-b)\lambda_1}{s+2R_1} [(1 + \lambda_{m_1} V_1(s))[P_0 + P_1]$$

$$= V_2(s)[P_0 + P_1] \tag{27}$$

where $V_2(s) = \frac{(1-b)\lambda_1}{s+2R_1}[(1 + \lambda_{m_1} V_1(s)$

from equation 8

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \lambda_{m_2} + \lambda_2 + \alpha + \gamma + \phi_{m_1}(y)\right] P_{2,m_1}(y,t) = 0$$

Taking Laplace transforms on both sides

$$\Rightarrow \frac{\frac{\partial}{\partial y} P_{2,m_1}(y,s)}{P_{2,m_1}(y,s)} = -\left[s + \lambda_{m_2} + \lambda_2 + \alpha + \gamma + \phi_{m_1}(y)\right]$$

Integrating 'y' between the units '0' to 'y'

$$\Rightarrow P_{2,m_1}(y,s) = P_{2,m_1}(0,s) e^{-[s+\lambda_{m_2}+\lambda_2+\alpha+\gamma]y - \int_0^y \phi_{m_1}(y)dy}$$

from equation 18

$$P_{2,m_1}(0,t) = \lambda_{m_1} P_{2,0}(t) + b\lambda_1 \int_0^\infty P_{0,m_1}(y,t)dy + b\lambda_1 \int_0^\infty P_{1,m_1}(y,t)dy + R_1 P_r(t)$$

Taking Laplace transforms on both sides

$$\therefore P_{2,m_1}(y,s) + V_4(s)P_{2,0}(s) + V_5(s)[P_{0,0}(s) + P_{1,0}(s)]$$

where $V_3(s) = \dfrac{1 - s_{m_1}(s + \lambda_{m_1} + \lambda_2 + \alpha + \gamma)}{s + \lambda_{m_2} + \lambda_2 + \alpha + \gamma}$ (28)

$$V_4(s) = V_3(s)\lambda_{m_1}$$

$$V_5(s) = b\lambda_1 \lambda m_1 V_1(s) + R_1 V_2(s)$$

From equation (10)

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \phi_{m_2}(z)\right] P_{1,m_2}(z,t) = 0$$

Taking Laplace transforms on both sides

$$\Rightarrow \frac{\frac{\partial}{\partial y} P_{1,m_2}(z,s)}{P_{1,m_2}(z,s)} = -\left[s + \phi_{m_2}(z)\right]$$

Integrating ‘z’ between the limits ‘0’ to ‘z’

$$P_{1,m_2}(z,s) = P_{1,m_2}(0,s) e^{-sz - \int_0^z \phi_{m_2}(z)dz}$$

From equation 19

$$P_{1,m_2}(0,t) = \lambda_{m_2} P_{1,0}(t) + \lambda_{m_2} \int_0^\infty P_{1,m_1}(y,t)dy$$

Taking Laplace transforms on both sides

$$\therefore P_{1,m_2}(z,s) = [\lambda_{m_2} P_{1,0}(s) + \lambda_{m_2} \int_0^\infty P_{1,m_1}(y,s)dy] e^{-sz - \int_0^z \phi_{m_2}(z)dz}$$

$$P_{1,m_2}(z,s) = \lambda_{m_2}[1 + \lambda_{m_1} V_1(s)] V_6(s) P_{1,0}(s)$$
$$= V_7(s) P_{1,0}(s) \tag{29}$$

where $V_6(s) = \dfrac{1 - s_{m_2}(s)}{s}$

$$V_7(s) = \lambda_{m_2}[1 + \lambda_{m_1} V_1(s)] V_6(s)$$

From equation (12)

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial \lambda} + 2\phi_{m_2}(z)\right] P_{2,m_2}(z,t) = 0$$

Taking Laplace transforms on both sides

$$\Rightarrow \frac{\frac{\partial}{\partial y} P_{2,m_2}(z,s)}{P_{2,m_2}(z,s)} = -\left[s + 2\phi_{m_2}(z)\right]$$

Integrating ‘z’ between the limits ‘0’ to ‘z’

$$\Rightarrow P_{2,m_2}(z,s) = P_{2,m_2}(0,s) e^{-sz - \int_0^z \phi_{m_2}(z)dz}$$

From equation 21.

$$P_{2,m_2}(0,t) = \lambda_{m_2} P_{2,0}(t) + \lambda_{m_2} \int_0^\infty P_{2,m_2}(y,t)dy$$

Taking Laplace transforms on both sides

$$\therefore P_{2,m_2}(z,s) = \left[ \lambda_{m_2} P_{2,0}(s) + \lambda_{m_2} \int_0^\infty P_{2,m_1}(y,s)dy \right] e^{-sz-2\int_0^z \phi_{m_2}(z)dz}$$

$$= \lambda_{m_2}[1+V_4(s)]V_8(s)P_{2,0}(s) + \lambda_{m_2} V_8(s)V_5(s)P_{1,0}(s)$$

$$+ \lambda_{m_2} V_5(s)V_8(s)P_{0,0}(s)$$

$$= V_9(s)P_2(s) + V_{10}(s)P_{1,0}(s) + P_{0,0}(s) \tag{30}$$

where $V_8(s) = \dfrac{1-2s_{m_2}(s)}{s}$

$$V_9(s) = \lambda_{m_2}(1+V_4(s))V_8(s)$$

$$V_{10}(s) = \lambda_{m_2} V_8(s)V_5(s)$$

From equation (12)

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial h} + 2\delta(h) \right] P_H(h,t) = 0$$

Taking Laplace transforms on both sides

$$\Rightarrow \frac{\frac{\partial}{\partial h} P_H(h,s)}{P_H(h,s)} = -[s + 2\delta(h)]$$

Integrating ‘h’ between the limits ‘0’ to ‘h’

$$\Rightarrow P_H(h,s) = P_H(0,s)e^{-sh-2\int_0^h \delta(h)dh}$$

From equation 22

$$P_H(0,t) = \beta P_{2,0}(t)$$

Taking Laplace transforms on both sides

$$P_H(0,t) = \beta P_{2,0}(t)$$

After simplification

$$P_H(h,s) = \beta V_{11}(s)P_{2,0}(s) \tag{31}$$

where $V_{11}(s) = \dfrac{1-2s\delta_h(s)}{s}$

from equation 13

$$\left[ \frac{\partial}{\partial t} + \frac{\partial}{\partial v} + 2\delta(v) \right] P_E(v,t) = 0$$

Taking Laplace transforms on both sides

$$\Rightarrow \frac{\frac{\partial}{\partial v} P_E(v,s)}{P_E(v,s)} = -[s + 2\delta(v)]$$

Integrating ‘v’ between the limits ‘0’ to ‘v’

$$\Rightarrow P_E(v,s) = P_E(0,s)e^{-sv-2\int_0^v \delta(v)dv}$$

From equation 14

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial u}+2\delta(u)\right]P_C(u,t)=0$$

Taking Laplace transforms on both sides

$$\Rightarrow \frac{\frac{\partial}{\partial v}P_C(u,s)}{P_C(u,s)}=-[s+\delta(u)]$$

Integrating ‘u’ between the limits ‘0’ to ‘u’

$$\Rightarrow P_C(u,s)=P_C(0,s)e^{-su-2\int_0^u \delta(u)du}$$

From equation 22

$$P_E(0,t)=\alpha P_{0,0}(t)=\alpha P_{1,0}(t)+\alpha P_{2,0}(t)+\alpha\int_0^\infty P_{0,m_1}(y,t)dy$$

$$+\alpha\int_0^\infty P_{1,m_1}(y,t)dy+\alpha\int_0^\infty P_{2,m_1}(y,t)dy$$

Taking Laplace transforms on both sides

$$P_E(0,s)=\alpha P_{0,0}(s)+\alpha P_{1,0}(s)+\alpha P_{2,0}(s)+\alpha\int_0^\infty P_{0,m_1}(y,s)dy$$

$$+\alpha\int_0^\infty P_{1,m_1}(y,s)dy+\alpha\int_0^\infty P_{2,m_1}(y,s)dy$$

After simplification

$$P_E(s)=\alpha[1+\lambda_{m_1}V_1(s)+V_5(s)]V_{12}(s)P_{0,0}(s)$$
$$+\alpha[1+\lambda_{m_1}V_1(s)+V_5(s)]V_{12}(s)P_{1,0}(s)$$
$$+\alpha[1+V_4(s)]V_{12}(s))P_{2,0}(s)$$

$$\Rightarrow P_E(s)=V_{13}(s)P_{2,0}(s)+V_{14}(s)[P_{0,0}(s)+P_{1,0}(s)] \tag{32}$$

where

$$V_{12}(s)=\frac{1-2s\delta(s)}{s}$$
$$V_{13}(s)=\alpha[1+V_4(s)]V_{12}(s)$$
$$V_{14}(s)=\alpha[1+\lambda_{m_1}V_1(s)+V_5(s)]V_{12}(s)$$

From equation 23

$$P_C(0,t)=\gamma P_{0,0}(t)+\gamma P_{1,0}(t)+\gamma P_{2,0}(t)+\gamma\int_0^\infty P_{0,m_1}(y,t)dy$$

$$+\gamma\int_0^\infty P_{1,m_1}(y,t)dy+\gamma\int_0^\infty P_{2,m_1}(y,t)dy$$

Taking Laplace transforms on both sides

$$P_C(0,s)=\gamma P_{0,0}(s)+\gamma P_{1,0}(s)+\gamma P_{2,0}(s)+\gamma\int_0^\infty P_{0,m_1}(y,s)dy$$

$$+\gamma\int_0^\infty P_{1,m_1}(y,s)dy+\gamma\int_0^\infty P_{2,m_1}(y,s)dy$$

After simplification

$$P_C(s) = \gamma[1+\lambda_{m_1}V_1(s)+V_5(s)]V_{12}(s)P_{0,0}(s)$$
$$+\gamma[1+\lambda_{m_1}V_1(s)+V_5(s)]V_{12}(s)P_{1,0}(s)$$
$$+\gamma[1+V_4(s)]V_{12}(s)P_{2,0}(s)$$

$$P_C(s) = V_{15}(s)P_{2,0}(s)a + V_{16}(s)[P_{0,0}(s)+P_{1,0}(s)] \tag{33}$$

where

$$V_{12}(s) = \frac{1-2s\delta(s)}{s}$$
$$V_{15}(s) = \gamma[1+V_4(s)]V_{12}(s)$$
$$V_{16}(s) = \gamma[1+\lambda_{m_1}V_1(s)+V_5(s)]V_{12}(s)$$

From equation 5

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\phi_B(x)\right]P_{3,0}(x,t)=0$$

Taking Laplace transforms on both sides

$$\Rightarrow \frac{\frac{\partial}{\partial x}P_{3,0}(x,s)}{P_{3,0}(x,s)} = -[s+2\phi_B(x)]$$

Integrating ‘x’ between the limits ‘0’ to ‘x’

$$\Rightarrow P_{3,0}(x,s) = P_{3,0}(0,s)e^{-sx-2\int_0^x \phi_B(x)dx}$$

From equation 14

$$P_{3,0}(0,t) = \lambda_2 P_{2,0}(t) + \int_0^\infty \lambda_2 P_{2,m_1}(y,t)dy$$

Taking Laplace transforms on both sides

$$P_{3,0}(0,s) = \lambda_2 P_{2,0}(s) + \int_0^\infty \lambda_2 P_{2,m_1}(y,s)dy$$

Substituting the value of $P_{3,0}(0,s)$ in the above equation we get

$$P_{3,0}(x,s) = V_{18}(s)P_{2,0}(s) + V_{19}(s)[P_{1,0}(s)+P_{0,0}(s)] \tag{34}$$

where

$$V_{17}(s) = \frac{1-2s_B(s)}{s}$$
$$V_{18}(s) = \lambda_2[1+V_4(s)]V_{17}(s)$$
$$V_{19}(s) = \lambda_2 V_5(s) + V_{17}(s)$$

From equation 3

$$\left[\frac{d}{dt}+\lambda_{m_1}+\lambda_{m_2}+\lambda_2+\alpha+\beta+\gamma\right]P_{2,0}(t) = b\lambda_1 P_{0,0}(t) + b\lambda_1 P_{1,0}(t)$$
$$+R_1 P_r(t) + \int_0^\infty P_{2,m_1}(y,t)\phi_{m_1}(y)dy$$

Taking Laplace transforms on both sides

$$\left[S+\lambda_{m_1}+\lambda_{m_2}+\lambda_2+\alpha+\beta+\gamma\right]P_{2,0}(s) = b\lambda_1 P_{0,0}(s) + b\lambda_1 P_{1,0}(s)$$
$$+R_1 P_r(s) + \int_0^\infty P_{2,m_1}(y,s)\phi_{m_1}(y)dy$$

Substituting the values of $P_r(s)$, $P_{2,m_1(y,s)}$ in the above equation we get

$$P_{2,0}(s) = V_{22}(s)[P_{0,0}(s) + P_{1,0}(s)] \tag{35}$$

$$V_{20}(s) = b\lambda_1 + R_1V_2(s) + V_5(s)S_{m_1}(s + \lambda_{m_2} + \lambda_{m_1} + \lambda_2 + \alpha + \beta + \gamma)$$

$$V_{21}(s) = (s + \lambda_{m_2} + \lambda_{m_1} + \lambda_2 + \alpha + \gamma)$$
$$- \lambda_{m_1}S_{m_1}(s + \lambda_{m_2} + \lambda_{m_1} + \lambda_2 + \alpha + \beta + \gamma)$$

$$V_{22}(s) = \frac{V_{20}(s)}{V_{21}(s)}$$

from equation 2

$$\left[\frac{d}{dt} + \lambda_{m_2} + \lambda_{m_1} + \lambda_1 + \alpha + \lambda\right]P_{1,0}(t) = \int_0^\infty P_{3,0}(x,t)\phi_B(x)dx$$
$$+ \int_0^\infty P_{1,m_1}(y,t)\phi_{m_1}(y)dy + \int_0^\infty P_{1,m_2}(z,t)\phi_{m_2}(z)dz$$
$$+ \int_0^\infty P_E(v,t)\delta(v)dx + \int_0^\infty P_{2,m_2}(z,t)\phi_{m_2}(z)dz + \int_0^\infty P_H(h,t)\delta(h)dx$$
$$+ \int_0^\infty P_C(u,t)\delta(u)dx$$

Taking Laplace transforms on both sides

$$\left[S + \lambda_{m_2} + \lambda_{m_1} + \lambda_1 + \alpha + \gamma\right]P_{1,0}(s) = \int_0^\infty P_{3,0}(x,s)\phi_B(x)dx$$
$$+ \int_0^\infty P_{1,m_1}(y,s)\phi_{m_1}(y)dy + \int_0^\infty P_{1,m_2}(z,s)\phi_{m_2}(z)dz + \int_0^\infty P_E(v,s)\delta(v)dx$$
$$+ \int_0^\infty P_{2,m_2}(z,s)\phi_{m_2}(z)dz + \int_0^\infty P_H(h,s)\delta(h)dx + \int_0^\infty P_C(u,s)\delta(u)dx$$

Substituting the values of
$P_{3.0}(x,s)$, $P_{1.m_1}(y,s)$, $P_{1.m_2}(z,s)$, $P_{2.m_2}(z,s)$, $P_E(v,s)$, $P_H(h,s)$, $P_C(u,s)$
In the above equation we get

$$P_{1,0}(s) = V_{26}(s)P_{2,0}(s) + V_{27}(s)P_{0,0}(s) \tag{36}$$

where

$$V_{23}(s) = \lambda_2(1 + V_4(s)S_B(s) + \alpha[(1 + V_4(s)]S\delta(s)$$
$$+ \gamma[(1 + V_4(s)]S\delta(s) + \lambda_{m_2}[(1 + V_4(s)]S_{m_2}(s) + \beta S\delta(s)$$

$$V_{24}(s) = \lambda_2V_5(s)S_B(s) + \alpha[1 + \lambda_{m_1}V_1(s) + V_5(s)]S\delta(s) + \gamma[1 + \lambda_{m_1}V_1(s)$$
$$+ V_5(s)]S\delta(s) + \lambda_{m_2}V_8(s)S_{m_2}(s)$$

$$V_{25}(s) = (S + \lambda_{m_2} + \lambda_{m_1} + \lambda_1 + \alpha + \gamma) - V_{24}(s) - \lambda m_1 sm_1(s) - \lambda_{m_2}[1 + \lambda_{m_1}V_1(s)]S_{m_2}(s)$$

$$V_{26}(s) = \frac{V_{23}(s)}{V_{25}(s)} \qquad V_{27}(s) = \frac{V_{24}(s)}{V_{25}(s)}$$

From equation 1

$$\left[\frac{d}{dt}+\lambda_{m_2}+\lambda_{m_1}+\lambda_1+\alpha+\gamma\right]P_{0,0}(t)=\int_0^\infty P_{3,0}(x,t)\phi_B(x)dx$$
$$+\int_0^\infty P_{0,m_1}(y,t)\phi_{m_1}(y)dy+\int_0^\infty P_{0,m_2}(z,t)\phi_{m_2}(z)dz$$
$$+\int_0^\infty P_E(v,t)\delta(v)dx+\int_0^\infty P_{2,m_2}(z,t)\phi_{m_2}(z)dz+\int_0^\infty P_H(h,t)\delta(h)dx$$
$$+\int_0^\infty P_C(u,t)\delta(u)dx$$

Taking Laplace transforms on both sides

$$[S+\lambda_{m_2}+\lambda_{m_1}+\lambda_1+\alpha+\gamma]P_{0,0}(s)=\int_0^\infty P_{3,0}(x,s)\phi_B(x)dx$$
$$+\int_0^\infty P_{0,m_1}(y,s)\phi_{m_1}(y)dy+\int_0^\infty P_{0,m_2}(z,s)\phi_{m_2}(z)dz+\int_0^\infty P_E(v,s)\delta(v)dx$$
$$+\int_0^\infty P_{2,m_2}(z,s)\phi_{m_2}(z)dz+\int_0^\infty P_H(h,s)\delta(h)dx+\int_0^\infty P_C(u,s)\delta(u)dx$$

Substituting the values of

$P_{3.0}(x,s)$, $P_{0.m_1}(y,s)$, $P_{0.m_2}(z,s)$, $P_{2.m_2}(z,s)$, $P_E(v,s)$, $P_H(h,s)$, $P_C(u,s)$ in the above equation we get

$$P_{25}(s)P_{0,0}(s)=V_{23}(s)P_{2,0}(s)+V_{24}(s)P_{1,0}(s)+1 \tag{37}$$

From equation 34

$$P_{2,0}(x,s)=V_{22}(s)[P_{0,0}(s)+P_{1,0}(s)]$$

Substitute the value of $P_{1,0}(s)$ in this equation we get

$$P_{2,0}(s)=V_{22}(s)P_{0,0}(s)+V_{22}(s)V_{26}(s)\,\mathrm{P}_{2,0}(s)+V_{22}(s)V_{27}(s)\mathrm{P}_{0,0}(s)$$
$$\Rightarrow P_{2,0}(s)=\frac{V_{22}(s)[1+V_{27}(s)]}{1-V_{22}(s)V_{26}(s)}P_{0,0}(s)$$
$$\Rightarrow P_{2,0}(s)=\frac{V_{29}(s)}{V_{28}(s)}P_{0,0}(s) \tag{38}$$

where $V_{28}(s)=1-V_{22}(s)V_{26}(s)$

$$V_{29}(s)=V_{22}(s)[1+V_{27}(s)]$$

From equation 36

$$P_{1,0}(s)=V_{26}(s)P_{2,0}(s)+V_{27}(s)P_{0,0}(s)$$

Substitute the vale of $P_{2,0}(s)$ in this equation we get

$$\Rightarrow P_{1,0}(s)=\frac{V_{26}(s)V_{22}(s)+V_{27}(s)}{1-V_{26}(s)V_{22}(s)}P_{0,0}(s)$$
$$\Rightarrow P_{1,0}(s)=\frac{V_{30}(s)}{V_{28}(s)}P_{0,0}(s)$$

where

$$V_{30}(s)=V_{26}(s)V_{22}(s)+V_{27}(s)$$

From equation 37

$$P_{25}(s)P_{0,0}(s)=V_{23}(s)P_{2,0}(s)+V_{24}(s)P_{1,0}(s)+1$$

Substitute the value of $P_{2,0}(s), P_{1,0}(s)$ in this equation we get

$$\Rightarrow P_{0,0}(s) = \frac{V_{28}(s)}{V_{31}(s)} \tag{39}$$

where $V_{31}(s) = V_{25}(s)\mathrm{P}_{0,0}(s) - V_{23}(s)V_{29}(s) - V_{24}(s)V_{30}(s)$

Substituting the value $\mathrm{P}_{0,0}(s)$ in equation (37, 38) we get

$$P_{1,0}(s) = \frac{V_{30}(s)}{V_{31}(s)} \tag{40}$$

$$P_{2,0}(s) = \frac{V_{29}(s)}{V_{31}(s)} \tag{41}$$

**Evaluation of operational Availability and Non-Availability**

The Laplace transforms of the probability that the system is in operable up and down state at time 't' can be evaluated as follows.

$$\begin{aligned} P_{UP}(s) &= P_{0,0}(s) + P_{1,0}(s) + P_{2,0}(s) + P_{0,m_1}(s) + P_{1,m_1}(s) + P_{2,m_1}(s) \\ &= [1+\lambda_{\mathrm{m}_1}V_1(s) + V_5(s)]P_{0,0}(s) + [1+\lambda_{\mathrm{m}_1}V_1(s) + V_5(s)]P_{1,0}(s) \\ &\quad + [1+V_4(s)]P_{2,0}(s) \end{aligned} \tag{42}$$

$$\begin{aligned} P_{dawn}(s) &= P_{3,0}(s) + P_{0,m_2}(s) + P_{1,m_2}(s) + P_{2,m_2}(s)P_E(s) + P_r(s) + P_H(s) + P_C(s) \\ &= [V_7(s) + V_{10}(s) + V_{14}(s) + V_{16}(s) + V_{19}(s)](P_{0,0}(s) + P_{1,0}(s)) \\ &\quad + [V_9(s) + V_{11}(s) + V_{13}(s) + V_{15}(s) + V_{18}(s)](P_{2,0}(s) \end{aligned} \tag{43}$$

**ERGODIC BEHAVIOUR**

Using Abel's lemma is Laplace transform,

viz., $\underset{s \to 0}{lt}\ \overline{sf(s)} = \underset{t \to 0}{lt}\ f(t) = f(say)$, provided that the limit on the R.H.S exists, the time independent up and down state probabilities are as follows.

$$\begin{aligned} P_{UP}(s) &= P_{0,0}(s) + P_{1,0}(s) + P_{2,0}(s) + P_{0,m_1}(s) + P_{1,m_1}(s) + P_{2,m_1}(s) \\ &= [1+\lambda_{\mathrm{m}_1}V_1(0) + V_5(0)]\frac{V_{28}(0)}{V_{31}(0)} + [1+\lambda_{\mathrm{m}_1}V_1(0) + V_5(0)]\frac{V_{30}(0)}{V_{31}(0)} \\ &\quad + [1+V_4(0)]\frac{V_{29}(0)}{V_{31}(0)} \end{aligned} \tag{44}$$

$$\begin{aligned} P_{down}(s) &= P_{3,0}(s) + P_{0,m_2}(s) + +P_{1,m_2}(s) + +P_{2,m_2}(s)P_E(s) + P_r(s) + P_H(s) + P_C(s) \\ &= [V_7(0) + V_{10}(0) + V_{14}(0) + V_{16}(0) + V_{19}(0)]\left(\frac{V_{28}(0)}{V_{31}(0)} + \frac{V_{30}(0)}{V_{31}(0)}\right) \\ &\quad [V_9(0) + V_{11}(0) + V_{13}(0) + V_{15}(0) + V_{18}(0)]\left[\frac{V_{29}(0)}{V_{31}(0)}\right] \end{aligned} \tag{45}$$

**PARTICULAR CASE:-** When repair follows exponential distribution setting $S_i(s) = \dfrac{\phi_i}{s+\phi_i}$ and $S_\delta(s) = \dfrac{\delta}{s+\delta}$ where i = B, $m_1$, $m_2$ in results (25) to (34) (39) (40) (41) one may get the various probabilities as follows:

$$P_{0,0}(s)=\frac{g_{28}(s)}{g_{31}(s)}$$

$$P_{1,0}(s)=\frac{g_{30}(s)}{g_{31}(s)}$$

$$P_{2,0}(s)=\frac{g_{29}(s)}{g_{31}(s)}$$

$$P_{3,0}(s)=g_{18}(s)P_{2,0}(s)+g_{19}(s)[P_{0,0}(s)+P_{1,0}(s)]$$

$$P_{0,m_1}(s)=g_1(s)\lambda_{m_1}P_{0,0}(s)$$

$$P_{1,m_1}(s)=g_1(s)\lambda_{m_1}P_{1,0}(s)$$

$$P_r(s)=g_2(s)[P_{0,0}(s)+P_{1,0}(s)]$$

$$P_{2,m_1}(s)=g_4(s)P_{2,0}(s)+g_5(s)[P_{0,0}(s)+P_{1,0}(s)]$$

$$P_{0,m_2}(s)=g_7(s)P_{0,0}(s)$$

$$P_{1,m_2}(s)=g_7(s)P_{1,0}(s)$$

$$P_{2,m_2}(s)=g_9(s)P_{2,0}(s)+g_{10}(s)[P_{1,0}(s)+P_{0,0}(s)]$$

$$P_H(s)=g_{11}(s)\beta P_{2,0}(s)$$

$$P_E(s)=g_{13}(s)P_{2,0}(s)+g_{14}(s)[P_{0,0}(s)+P_{1,0}(s)]$$

$$P_C(s)=g_{15}(s)P_{2,0}(s)+g_{16}(s)[P_{0,0}(s)+P_{1,0}(s)]$$

where

$$g_{15}(s)=(S+\lambda_{m_2}+\lambda_1+\alpha+\gamma+\phi_{m_1})^{-1}$$

$$g_2(s)=\frac{(1-b)\lambda_1}{s+2R_1}[1+\lambda_{m_1}g_1(s)]$$

$$g_3(s)=(S+\lambda_{m_2}+\lambda_2+\alpha+\gamma+\phi_{m_1})^{-1}$$

$$g_4(s)=g_3(s)\lambda m_1$$

$$g_5(s)=b\lambda_1\lambda m_1g_1(s)+R_1g_2(s)$$

$$g_6(s)=(s+\phi_{m_2})^{-1}$$

$$g_7(s)=\lambda_{m_2}[1+\lambda_{m_1}g_1(s)]g_6(s)$$

$$g_8(s)=\frac{s-\phi_{m_2}}{s(s+\phi_{m_2})}$$

$$g_9(s)=\lambda_{m_2}[1+g_4(s)]g_8(s)$$

$$g_{10}(s)=\lambda_{m_2}g_5(s)]g_8(s)$$

$$g_{11}(s)=\frac{s-\delta_h}{s(s+\delta_h)}$$

$$g_{12}(s)=\frac{s-\delta}{s(s+\delta)}$$

$$g_{13}(s)=\alpha[1+g_4(s)]g_{12}(s)$$

$$g_{14}(s)=\alpha[1+\lambda_{m_1}g_1(s)+g_5(s)]g_{12}(s)$$

$$g_{15}(s)=\gamma[1+g_4(s)]g_{12}(s)$$

$$g_{16}(s) = \gamma[1+\lambda_{m_1} g_1(s) + g_5(s)]g_{12}(s)$$

$$g_{17}(s) = \frac{s-\phi_B}{s(s+\phi_B)}$$

$$g_{18}(s) = \lambda_2[1+g_4(s)]g_{17}(s)$$

$$g_{19}(s) = \lambda_2 g_5(s) g_{17}(s)$$

$$g_{20}(s) = b\lambda_1 + R_1 g_2(s) + g_5(s)\phi_{m_1}(s+\lambda_{m_2} + \lambda_2 + \alpha + \gamma + \beta)^{-1}$$

$$g_{21}(s) = (s+\lambda_{m_2} + \lambda_{m_1} + \lambda_2 + \alpha + \gamma)$$
$$-\lambda_{m_1}\phi_{m_1}(s+\lambda_{m_2} + \lambda_{m_1} + \lambda_2 + \alpha + \beta + \gamma)^{-1}$$

$$g_{22}(s) = \frac{g_{20}(s)}{g_{21}(s)}$$

$$g_{23}(s) = \lambda_2(1+g_4(s)+\phi_B(s)+\phi_B)^{-1} + \alpha[1+g_4(s)] + \delta(s+\delta)^{-1}$$
$$+\gamma(1+g_4(s)]\delta(s+\delta)^{-1} + \lambda_{m_2}[1+g_4(s)]\phi_{m_2}(s+\phi_{m_2})^{-1}$$
$$+\beta\delta(s+\delta)^{-1}$$

$$g_{24}(s) = \lambda_2 g_5(s) + \phi_B(s+\phi_B)^{-1} + \alpha[1+\lambda_{m_1} g_1(s) + g_5(s)]\delta(s+\delta)^{-1}$$
$$+\gamma[1+\lambda_{m_1} g_1(s) + g_5(s)]\delta(s+\delta)^{-1} + \lambda_{m_2} g_8(s)\phi_{m_2}(s+\phi_{m_2})^{-1}$$

$$g_{25}(s) = (s+\lambda_{m_2} + \lambda_{m_1} + \lambda_2 + \alpha + \gamma) - g_{24}(s)$$
$$-\lambda_{m_1}\phi_{m_1}(s+\phi_{m_1})^{-1} - \lambda_{m_2}[1+\lambda_{m_1} g_1(s)]\phi_{m_2}(s+\phi_{m_2})^{-1}$$

$$g_{26}(s) = \frac{g_{23}(s)}{g_{25}(s)}$$

$$g_{27}(s) = \frac{g_{24}(s)}{g_{25}(s)}$$

$$g_{28}(s) = 1 - g_{22}(s)g_{26}(s)$$

$$g_{29}(s) = g_{22}(s)[1+g_{27}(s)]$$

$$g_{30}(s) = g_{26}(s)g_{22}(s) + g_{27}(s)$$

$$g_{31}(s) = g_{25}(s)g_{28}(s) - g_{23}(s)g_{29}(s) - g_{24}(s)g_{30}(s)$$

**UP and Down state probabilities**

The Laplace transforms of up and down state probabilities are as follows.

$$P_{UP}(s) = P_{0,0}(s) + P_{1,0}(s) + P_{2,0}(s) + P_{0,m_1}(s) + P_{1,m_1}(s) + P_{2,m_1}(s)$$
$$= [1+\lambda_{m_1} g_1(s) + g_5(s)]P_{0,0}(s) + [1+\lambda_{m_1} g_1(s) + g_5(s)]P_{1,0}(s) \quad (44)$$
$$+[1+g_4(s)]P_{2,0}(s)$$

$$P_{down}(s) = P_{3,0}(s) + P_{0,m_2}(s) + P_{1,m_2}(s) + P_{2,m_2}(s) + P_E(s) + P_r(s) + P_H(s) + P_C(s)$$
$$= [g_7 + g_{10} + g_{14} + g_{16} + g_{19}]P_{0,0}(s)$$
$$+[g_7 + g_{10} + g_4 + g_{16} + g_{19}]P_{1,0}(s)$$
$$+[g_9 + g_{11} + g_{13} + g_{15} + g_{18}]P_{2,0}(s)$$

**RELIABILITY:**

Laplace transforms of the reliability of the system given as follows:

$$R(s)=\frac{1}{S+A}\left[1+\frac{\lambda_{m_1}}{s+C}+\frac{b\lambda_1\lambda_{m_1}}{s+C}+\frac{b\lambda_1}{s+B}+\frac{b\lambda_1\lambda_{m_1}}{(s+B)(s+D)}\right]$$

Taking inverse Laplace transforms of the above equation we get

$$R(t)=e^{-At}\left[1+\frac{\lambda_{m_1}}{C-A}+\frac{b\lambda_1}{B-A}+\frac{b\lambda_1\lambda_{m_1}}{(B-A)(D-A)}+\frac{b\lambda_1\lambda_{m_1}}{(C-A)}\right]$$
$$+e^{-Bt}\left[\frac{-b\lambda_1}{(B-A)}+\frac{b\lambda_1\lambda_{m_1}}{(A-B)(D-B)}\right]+e^{-Ct}\left[\frac{-\lambda m_1}{(C-A)}-\frac{b\lambda_1\lambda_{m_1}}{(C-A)}\right]$$
$$+e^{-Dt}\left[\frac{b\lambda_1\lambda_{m_1}}{(A-B)(B-D)}\right]$$

**M.T.T.F**

Mean time to system failure is the expected time to operate the system successfully which is given as follows:

$$MT.T.F=\int_0^\infty R(t)dt$$
$$=\frac{1}{A}\left[1+\frac{b\lambda m_1}{(B-A)}+\frac{\lambda m_1}{(C-A)}+\frac{b\lambda_1\lambda_{m_1}}{C-A}+\frac{b\lambda_1\lambda_{m_1}}{(D+A)(B-A)}\right]$$
$$+\frac{1}{B}\left[\frac{-b\lambda_1}{B-A}+\frac{b\lambda_1\lambda_{m_1}}{(D-B)(A-B)}\right]+\frac{1}{C}\left[\frac{-\lambda_{m_1}}{(C-A)}+\frac{b\lambda_1\lambda_{m_1}}{(A-C)}\right]$$
$$+\frac{1}{D}\left[\frac{b\lambda_1\lambda_{m_1}}{(B-D)(A-D)}\right]$$

where $A=\lambda_{m_1}+\lambda_{m_2}+\lambda_1+\alpha+\gamma$, $B=\lambda_{m_1}+\lambda_{m_2}+\lambda_1+\alpha+\beta+\gamma$

$C=\lambda_{m_2}+\lambda_1+\alpha+\gamma$, $D=\lambda_{m_2}+\lambda_2+\alpha+\gamma$

**Numerical Illustrations:**

**Reliability Analysis:**

$\lambda_1=0.01$, $\lambda_2=0.02$, $\lambda_{m_1}=0.011$, $\lambda_{m_2}=0.015$, $\alpha = 0.01$, $\beta = 0.02$, $\gamma = 0.03$, b = 0.96 and for different values of t in the equation (60) one may obtain the reliability of the system as given in fig 2. The reliability of the system decreases slowly as the time period increases. It also depicts the reliability of the system for a long time period.

**M.T.T.F**

$\lambda_1 = 0.01$, $\lambda_2 = 0.02$, $\lambda_{m_1}=0.01$, b = 0.96, $\beta = 0.02$ and taking different values of $\lambda_{m_2}$ in the equation (61) one may obtain the variations of M.T.T.F. of the system against the environmental failure rate $\alpha$ shown in figure 3.

The variations in M.T.T.F w.r.to Environmental failure rate as the major failure rate of the subsystem A increases. The series of curves Represents that MTTF decreases as the environmental failure increases apparently.

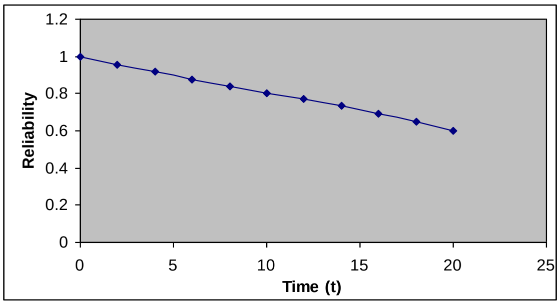



**Fig. 2**

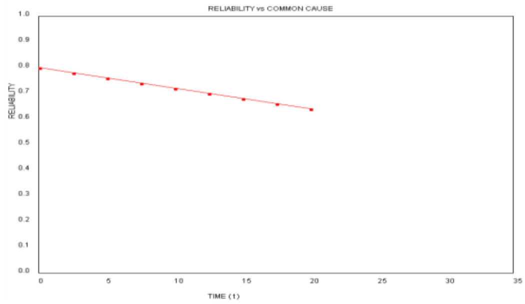



**Fig. 3**

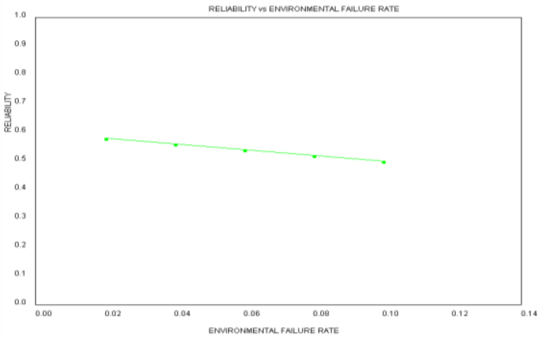
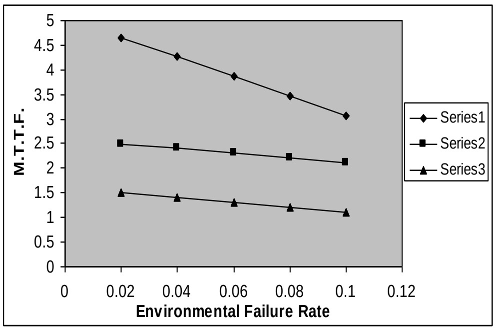



**Fig. 4**




**Series 1 2 3**

$\lambda_{m_1} = 0.2$ $\lambda_{m_2} = 0.4$ $\lambda_{m_3} = 0.6$

**Fig.5**

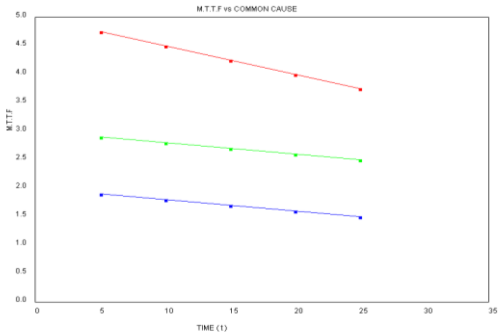



**Fig.6**

## V. DISCUSSION

In this paper we presented mathematical models of imperfect switching, environmental, common cause and human error effects, so probabilistic behaviour considering various values of coefficient of human error, common cause, environmental effects respectively.

## VI. CONCLUSIONS :

The Reliability, MTTF curves are plotted in figures (2), (3). From these graphs we observe that

i) The Reliability of the system decreases slowly as the time period increases.

ii) The MTTF decreases as the environmental failure increases

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